Appendix D

Useful Mathematical Formulas

D.1 SUMMATION FORMULAS

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

$$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \qquad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \qquad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n^2\alpha^n = \frac{\alpha^2+\alpha}{(1-\alpha)^3} \qquad |\alpha| < 1$$

D.2 EULER'S FORMULAS

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

D.3 TRIGONOMETRIC IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\cos^2\theta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \cos\alpha\cos\beta$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$a\cos\alpha + b\sin\alpha = \sqrt{a^2+b^2}\cos\left(\alpha - \tan^{-1}\frac{b}{a}\right)$$

D.4 POWER SERIES EXPANSIONS

$$e^{\alpha} = \sum_{k=0}^{\infty}\frac{\alpha^k}{k!} = 1 + \alpha + \frac{1}{2!}\alpha^2 + \frac{1}{3!}\alpha^3 + \cdots$$

$$(1+\alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!}\alpha^2 + \cdots + \binom{n}{k}\alpha^k + \cdots + \alpha^n$$

$$\ln(1+\alpha) = \alpha - \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 - \cdots + \frac{(-1)^{k+1}}{k}\alpha^k + \cdots \qquad |\alpha| < 1$$

D.5 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$$e^{\alpha}e^{\beta} = e^{\alpha+\beta}$$

$$\frac{e^{\alpha}}{e^{\beta}} = e^{\alpha-\beta}$$

$$\ln(\alpha\beta) = \ln\alpha + \ln\beta$$

$$\ln\frac{\alpha}{\beta} = \ln\alpha - \ln\beta$$

$$\ln\alpha^{\beta} = \beta\ln\alpha$$

$$\log_b N = \log_a N \log_b a = \frac{\log_a N}{\log_a b}$$

D.6 SOME DEFINITE INTEGRALS

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}} \qquad a > 0$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \qquad a > 0$$

$$\int_0^\infty x e^{-ax^2}\,dx = \frac{1}{2a} \qquad a > 0$$